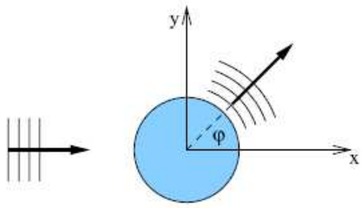
Figure 1: Schematic diagram of scattering process from a circular quantum dot.



1 Projects

1.1 Short project: Scattering from a magnetic flux

The fact that graphene electrons cannot be localised by a potential barrier as Schrödinger electrons are, implies that they will behave very differently when scattered from a potential. In fact, their behavior in some something as simple as scattering from a theta function potential $V = \Theta(r)$ (illustrated schematically in Fig. 1) shows some remarkably novel features, in this case the formation of a *caustic*, see Fig. 2. In ray optics a caustic is a line to which all light rays are tangents. If you look at a glass of water you will, in fact, often see a caustic looking very similar to the one shown in Fig. 2; it is the so-called nephroid caustic. This behavior is therefore much closer to what one would expect with light, and not at all what one would expect of electrons in a solid.

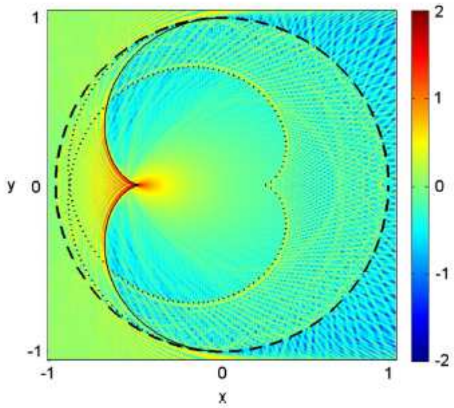
A short project would be to see what happens to the scattering properties when $V = V_0\Theta(r)$ is replaced by a magnetic field, i.e. $B = B_0\Theta(r)$. This is interesting because, unlike photons, electrons have a charge and so respond to a magnetic field through its vector potential $\mathbf{p} \to (\mathbf{p} + \frac{e}{c}\mathbf{A})$. Hence, here we are mixing the light-like and particle-like natures of graphene electrons.

The essence of the problem is to solve the Dirac-Weyl equation inside and outside the quantum dot using polar coordinates, and then match at the boundary (since the Dirac-Weyl equation is first order one needs only match the wavefunction value, not derivative as in the second order Schrödinger equation).

1.2 Longer project: Spectrum of the $1/r$ potential in presence of external magnetic field

In contrast to the case of the Schrödinger equation, the $1/r$ potential in Dirac-Weyl equation does not form bound states. For the Schrödinger equation a $1/r$ potential in an external magnetic field cannot be solved exactly. However,

Figure 2: Square of the Dirac-Weyl wavefunction inside the quantum dot. Thick dashed line is the dot boundary. Features inside the dot are caustics.



there are indications it can be solved for the Dirac-Weyl equation. (Both the cases uniform B with no potential, and $V = 1/r$ and $B = 0$, lead to the same differential equation). The question that could be answered, if an exact solution can be found, is whether or not there is a discrete spectrum and bound states even in the Dirac-Weyl equation with an external magnetic field. This is interesting since, as has been discussed above, localization in the Dirac-Weyl equation is difficult, and this could be a simple model of how one might trap graphene electrons (i.e. form bound states) on an impurity in the presence of a magnetic field.

1.3 Longer project: graphene band structure engineering

The novel band structure of graphene has led to some (even more) novel attempts to modify this band structure by changing, for example, the conic spectrum to that of an ellipsoidal spectrum. This was achieved with the use of a periodic external potential. Obviously, from a technological point of view controlling the properties of graphene is very interesting and, from a fundamental point of view, learning how the Dirac-Weyl equation behaves under various external potentials is also interesting. While scalar potentials have been extensively studied, may other possibilities exist (e.g. various gauge fields that may be generated not by an external magnetic field but by local distortions of the lattice), and this project would entail exploring the solutions of the Dirac-Weyl equation under these conditions.